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SOLAR WIND FLOW PAST COMETS

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SUMMARY

The cometary nucleus is considered in the hydrodynamic sense as a source located in a supersonic stationary plasma flow from the Sun. At the boundary of the divide between the source's and stream's media the pressure is defined by the Newtonian formula.

It is possible to explain the plasma head configuration and the cylindrical shape of the type-I tail. The shock wave recession is estimated by comparing with the solution for a source located in an incompressible medium.

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It has been generally admitted lately that the type-I cometary tails are conditioned by comet atmosphere interaction with the solar corpuscular streams. It is noted at the same time that the ionized tails are linked, first of all, with the quasistationary component of solar corpuscular radiation, that is, the solar wind [1]. This is why it appears to be possible to subdivide the events in type-I tails and in plasma heads into two groups — the stationary and nonstationary.

The stationary events are conditioned by the solar wind; they would also take place in the case when the wind itself is strictly stationary. Referred to here are the configurations of the head and the cylindrical shape of the tail. The contracting shells, the flapping rays and the acceleration of nebula formations must also be related to stationary events, or, to be more precise, to statistically stationary events (according to the above characteristics).

The nonstationary events are conditioned by periodical corpuscular streams, by their structure and partly by the nonstationary state of the wind [3]. The latter may be represented as a certain "modulating factor"

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\* OBTSEKANIYE KOMETY SOLNECHNYM VETROM.

superimposing itself upon a statistically stationary background. Comet flares, correlating with geoeactive streams are in particular referred to nonstationary events.

It will be shown below that the configuration of the head and the cylindrical shape of the tail may be explained by considering the stationary solar wind flow past the comet.

At solar wind interaction with the comet the length of the free path of ions is great: it is comparable with the characteristic dimension of the comet. But the solar wind carries along a weak magnetic field, and there is every reason to assume that the field in comets is not weaker than the interplanetary field [2]. What should be compared in the case of a magnetic field with a characteristic dimension is the gyroradius\* of ions and not the free path; and we may consider that in scales greater by comparison with it, the plasma is a continuum [2, 4]. Thus, for a field  $5 \cdot 10^{-5}$  gauss (the mean value in the solar wind) at a velocity of  $3 \cdot 10^7$  cm/sec the gyroradius is  $6 \cdot 10^7$  cm, while the characteristic dimension of the comet — the diameter of the head — is  $10^{10} - 10^{11}$  cm. Besides, according to [4], for the computation of solar wind flow past the Earth's magnetosphere, whose characteristic dimension coincides with that of a large comet, standard gas dynamics method may be applied provided we assume  $\gamma = 2$  (the particle motion in the magnetic field has two degrees of freedom); as to the pressure, we may understand the sum

$$p^* + \frac{H^2}{8},$$

where  $p^*$  is the gas pressure. The Mach number  $M$  is substituted by the Alfvén Mach number

$$M = V / V_A,$$

where  $V$  is the velocity of the unperturbed flow and  $V_A$  is the Alfvén velocity in the unperturbed flow.

In the current note we shall forego the consideration of the specific effects of the magnetic field, for we consider that by ascribing it the state of "continuum", its role is being limited.

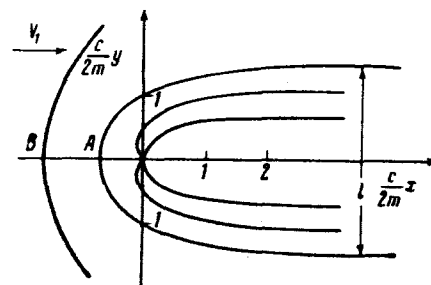


Fig. 1

\* the word gidroradius" used in the original text seems to be a misprint for "gyroradius", it is thus interpreted accordingly.

The ion component of the cometary atmosphere emerges apparently in a certain region surrounding the hard nucleus, and we may consider this region as the "source zone".

In the first approximation we shall estimate that the cometary plasma flow takes place at constant density and the source of ions is a point source situated in the comet's nucleus.

A supersonic flow passes around the source (for the solar wind  $M_A \sim 8$ ) and it is quite probable that a stationary collisionless shock wave forms at the same time from the comet's head upward along the flow [5].

Following are the considerations leading to such an assumption. The comet's atmosphere constitutes a continuum, so that the solar wind must flow past it as if it were a certain obstacle, of which the dimension coincides with the characteristic dimension of the comet. But the diameter of the head of a large comet is of the order of  $10^{10} - 10^{11}$  cm, that is, equal or greater than the characteristic dimension of the Earth's magnetosphere; consequently, for the solar wind the comet is an obstacle comparable with the Earth. Therefore, there must exist ahead of a large comet a shock wave analogous to the shock wave ahead of the magnetosphere.

The problem of a high supersonic flow of compressed gas past the source is in many traits comparable with the problem of a flow past a blunt body, so that its exact solution cannot, at present, be obtained in a general form and an approximate solution must be sought for.

The existing methods for resolving the problems of flow past axis-symmetrical and flat bodies may be subdivided into two groups. In the first group the nonstationary problem is solved and then the solutions for great  $t$  are considered, whereas in the second group the qualitative motion pattern is postulated [6]. In the problem about the source it is preferable to postulate the latter, for this method is simpler and it gives results agreeing well with experiment.

We shall start from the following assumption: an outgoing shock wave is forming upward along the flow from the source; the gas, ejected by the source, is bounded by a certain surface, of which the shape is determined by the equality of pressure of two media [Fig. 1].

It is proposed to conduct in the following the total solution of this problem. For the moment we shall limit ourselves to an approximate consider-

ation for which we shall make use of the Newton theory in the first approximation.

According to this approximation the surface of the jump coincides with that of the body or the surface, bounding in our case the gas source. As is shown by experiments on the flow past axis-symmetrical and plane bodies, the best agreement of computed data with the experimental is provided by the modified Newton formula for the pressure on body's surface [7]:

$$p = \frac{1}{2} \rho_1 V_1^2 c_p^* \sin^2 \alpha,$$

where  $\alpha$  is the inclination angle of surface element to the direction of the incident flow;  $\rho_1$  and  $V_1$  are respectively the density and the velocity of the flow. The factor  $c_p^*$  for blunt bodies is a function of the Mach number and of isentrope indicator  $\gamma$  [7]. Its variation region is not great; it oscillates within the limits 1.25 - 2.00 for  $M$  varying from 1 to  $\infty$  and  $\gamma$  - from 1 to  $5/3$ . For  $M = 8$  and  $\gamma = 2$  (solar wind)  $c_p^* = 1.67$ . Denoting the pressure in the forward part of the surface, where the shock wave is perpendicular, by  $p_0$ , we obtain:

$$p = p_0 \sin^2 \alpha, \quad (1)$$

where  $p_0 = 0.84 \rho_1 V_1^2$ .

For a flow with constant density the approximate solution may be obtained by the method given in the work [8] for the calculation of plasma flow past a linear current. We shall apply here this method only for the plane problem, which is precisely what we shall be assuming in the following.

Let us now postulate that in the plane  $z$ , where the true flow takes place, the source is at the origin of the coordinates, the source's gas occupies a region  $G$ , bounded by the curve  $S$ , which is the current line. Let the analytical function  $z = z(\xi)$  materialize the conformal transformation of the interior of a unitary radius circle with the center at the origin of the coordinates in the plane  $\xi$  to the region  $G$  in such a fashion that the point  $z = 0$  correspond to the point  $\xi = 0$ . In this case, to the complex flow potential in the region  $G$  -  $W(z)$  will correspond the complex flow potential in the unitary circle -  $W(\xi)$ , of which the following is known:

- a) the source must be located at the center of the circle;
- b) the circumference of unitary radius must be the current line;
- c) there must be either inside the circle or on the circumference a discharge, equal in power to the source, for in the opposite case there can be no stationary flow;
- d) at  $z(\zeta)$  transformation the point, at which the discharge outlet is located, must drift into an infinitely remote point.

Let us consider the potential

$$W = \ln \frac{(\zeta - 1)^{2m}}{\zeta^m}. \quad (2)$$

$W$  constitutes the potential of the source of power  $m$  at coordinate origin and of the discharge at point  $\zeta(1.0)$  of double power. The entire flow of the source's medium is concentrated inside the circumference of unitary radius constituting the current line. In this way the potential (2) satisfies the conditions a, b, c. That it satisfies also the condition d, is something that will be ascertained from the following.

We shall seek a function  $z = z(\zeta)$ , materializing the conformal transformation of the interior of the circle of unitary radius to the region  $G$ . The curve  $S$  in the plane  $z$  is determined by the equality of pressures of source's gas and incident flow. Making use of this, we shall obtain from (1) and the Bernoulli equation the boundary condition for the flow velocity of source's medium

$$V^2 = 2 \frac{\bar{p}_0}{\rho_w} \cos^2 \alpha, \quad (3)$$

where  $\rho_w$  is the source's gas density. The velocity in the plane  $z$  is linked with that in the plane  $\zeta$  as follows:

$$|V(z)| = \left| \frac{dw}{d\zeta} \right| \left| \frac{dz}{d\zeta} \right|.$$

Taking into account the boundary condition (3), we shall obtain:

$$\left| \frac{dw}{d\zeta} \right| = \pm c \cos \alpha \left| \frac{dz}{d\zeta} \right| \quad \text{where} \quad c = \sqrt{\frac{2p_0}{\rho_w}}.$$

The given correlation refers to the region's boundary, and this is why

$$|d\zeta| = d\varphi \quad \text{and} \quad \cos \alpha |dz| = dx.$$

We find the derivative  $d\psi/d\zeta$  from (2). Substituting these values into the preceding equality and integrating, we obtain

$$\ln \frac{1}{2}(1 - \cos \varphi) + c_1 = -\frac{c}{m}x. \quad (4)$$

The sign minus in the right-hand part follows from the normalization: at  $\varphi \rightarrow 0, x \rightarrow \infty$  since we estimate the flow velocity as directed toward the side of  $x$  increase;  $c_1$  is the integration constant.

The equality (4) gives the value of the real part of the function sought for on the circumference of unitary radius. Since the real and imaginary parts of the analytical function are conjugated harmonic functions, the imaginary part of  $z = z(\zeta)$  on the circumference may be found as follows: let us take the expansion [9]

$$\frac{1}{2} \ln \frac{1}{2(1 - \cos \varphi)} = \sum_{k=1}^{\infty} \frac{\cos k\varphi}{k}$$

Now the equality (4) may be written

$$\frac{c}{2m}x = \sum_{k=1}^{\infty} \frac{\cos k\varphi}{k} + \ln 2 - \frac{1}{2}c_1.$$

The conjugated harmonic functions are determined with a precision to the constant addend and this is why

$$\frac{c}{2m}y = \sum_{k=1}^{\infty} \frac{\sin k\varphi}{k} + c_2 = \frac{\pi - \varphi}{2} + c_2. \quad (5)$$

From considerations of symmetry we shall assume that at  $\varphi = \pi, y = 0$ , whence it follows  $c_2 = 0$ . The value of the constant in (4) is determined from the Schwartz integral by the requirement  $z = 0$  at  $\zeta = 0$ . Computing the integral we shall obtain  $c_1 = 2 \ln 2$ . Taking this into account and excluding  $\varphi$  from (4) and (5), we shall find the curve bounding the source's gas

$$\frac{c}{2m}x = -\ln \cos \frac{c}{2m}y - \ln 2. \quad (6)$$

Comparing this curve with the curve (9) from [8], we see that they coincide; this apparently may be explained by identical symmetry of the vortex of the magnetic field and the hydrodynamic source.

If the comet is treated as a source, the curve (6) gives the configuration of the head. It is however customary to estimate that the contour of the head is described by a catenary [2]. Let us represent the formula (6) and the catenary in the form of series:

$$x = -\frac{2m}{c} \ln \cos \frac{c}{2m} y - 2 \ln 2 \frac{m}{c} = -2 \ln 2 \frac{m}{c} + \frac{c}{4m} y^2 + \frac{c^3}{96m^3} y^4 + \dots;$$

$$x = -a \operatorname{ch} \frac{y}{a} = a + \frac{1}{2a} y^2 + \frac{1}{8a^3} y^4 + \dots$$

We see that the second and the third terms of the expansion coincide almost exactly (the first is immaterial, for it denotes the transfer along the axis  $x$ ). Therefore, the curve (6) satisfies the observation data: it describes the configuration of the comet's head and explains the cylindrical shape of the tail [11].

It should be noted that the catenary for the configuration of the comet's head was obtained in the work [12], where the nonstationary problem was resolved in the assumption that the contour of the head is determined by the shape of the magnetic lines of force.

The curve (6) bears the designation of "catenary of equal resistance". The function, materializing the conformal transformation of the interior of unitary radius circle to the interior of this curve, will be written

$$z = \frac{2m}{c} \ln \frac{1}{1-\xi}.$$

Eliminating  $\xi$  from the potential (2) with the aid of the formula we shall find the source's gas flow potential in the plane  $z$ :

$$w = -\frac{c}{2} z - m \ln (e^{cz/2m} - 1).$$

Hence it is easy to find the equation of current lines:

$$\frac{c}{2m} x = -\ln \left[ \cos \frac{c}{2m} y + \frac{\sin \frac{c}{2m} y}{\operatorname{tg} \left( \frac{\Psi}{m} + \frac{c}{2m} y \right)} \right],$$

where  $\Psi$  is only a function of the current's line. It is evident that at

$$\frac{\Psi}{m} = n\pi \quad (n = 0, 1, 2, \dots)$$

we shall obtain the boundary, that is, the curve (6). The lines of current

$$\frac{\Psi}{m} = \pm 0.25\pi \quad \text{and} \quad \frac{\Psi}{m} = \pm 0.5\pi$$

are plotted in Fig. 1.

The diameter of the head or, which is the same, the width of the tail, constitute the comet's characteristic dimension. It is evident that this is the distance between the asymptotes of current's boundary line ( $l$  in Fig. 1), equal to  $(2m/c)\pi$ . It is possible to estimate the dependence of tail's width on the quantity of gas emitted in the nucleus. We shall denote the latter quantity by  $Q$ ; it is evident that  $Q \sim ml$  and that consequently  $l \sim \sqrt{Q}$ .

We can not find the distances of jump discontinuity recession in the Newtonian approximation (AB in Fig. 1). However, we may estimate this quantity approximately as follows.

Assume that behind the shock wave the flow takes place with constant density. Such an admission is often made in the theory of hypersonic flows. Of interest to us will be only the region near the "nose" of the comet (A), that is, the region of small  $y$ , where the shock wave is nearly perpendicular.

Let us take for the solar wind the following data:  $H \sim 5 \cdot 10^{-5}$  gauss, the gas pressure is of the order of the magnetic, the velocity and density are respectively  $3 \cdot 10^7$  cm/sec and  $5 \text{ cm}^{-3}$ . It is easy to calculate that at the same time the pressure  $p_1$  is by one order smaller than the term  $\rho_1 V_1^2$ , this is why we postulate  $p_1 = 0$ . Then for a perpendicular shock wave we shall have

$$\frac{\rho_f}{\rho_1} = \frac{\gamma + 1}{\gamma - 1} = 3, \quad V_f = \frac{\rho_1 V_1}{\rho_f} = \frac{V_1}{3}, \quad (7)$$

where the index 1 refers to the solar wind (which, for brevity, we shall refer to as flow) prior to the passage of the shock wave, the index f — after its passage.

We shall neglect the distortion of the shock wave in the region of small  $y$ , and consequently, the flow's vorticity.

We shall consider a source of power  $\underline{m}$ , placed in the flow of an incompressible fluid, uniform at infinity. The complex potential of the source

in the flow has the form [10]

$$w_1 = -Uz - m \ln z,$$

where  $U$  is the flow velocity at infinity. The potential  $w_1$  is related to the case, where the fluids of the source and of the flow have identical densities, but in a physical problem this can not be fulfilled. Thus, in particular, the density of the cometary gas is substantially greater than that of the incident flow. In this case the potentials for the flow and for the source will be respectively written as

$$w_f = -\alpha z \frac{m}{\alpha} \ln z \quad \text{and} \quad w_s = -\alpha U z - m \ln z,$$

where  $\alpha$  is a constant, whose value is found from the condition of pressure equality of the two media at the boundary of the divide.

In the Bernoulli equation

$$\frac{p}{\rho} + \frac{1}{2} V^2 = C$$

the constants must have different values for the source and for the flow, for in the opposite case it may be demonstrated that the densities of the fluids are equal. At the point A (Fig. 1) the velocities are zero, and we may obtain from the equality of pressures

$$C_f \rho_f = C_s \rho_s.$$

At any point of the boundary of the divide  $p_w = p_f$  and from these equalities and the Bernoulli equation we shall obtain

$$\alpha = (\rho_f / \rho_s)^{1/2}.$$

From the potential  $w$  or  $w_f$  we may find the line bounding the fluid of the source:

$$x = -y \operatorname{ctg} \frac{\alpha U}{m} y$$

(See ref. [10]). It is easy to see that this curve coincides nearly exactly with the curve (6) for small  $y$  and on the condition  $\alpha U = c/2 \ln 2$ , that is, the comet's configuration near the "nose" agrees with the one that would occur at flow past it by an incompressible fluid, uniform at infinity. Consequently, it may be concluded that the shock wave settles at the distance over which the the velocity of the incompressible flow coincides with the velocity  $V_f$  of (7).

The velocity of the incompressible flow past the source is

$$\left| \frac{dw_f}{dz} \right| = \left| U + \frac{m}{az} \right|$$

and at  $y = 0$  we shall obtain  $b = m/\alpha (U - V_f)$ , where  $b$  is the distance between the source and the shock wave. We shall postulate  $U = c/2 \ln 2\alpha$  and  $\underline{m}$ , that is the power of the source, may be expressed for the comet's nucleus by the width of the tail  $l$ , utilizing formula (6):  $m = \frac{cl}{2\pi}$ . (\*)

The distance of the jump discontinuity recession is  $b - a$ , where  $a = 2m \ln 2 / c$  (OA in Fig.1). For the above-assumed properties of the flow

$$\alpha = \left( \frac{3\rho_1}{\rho_w} \right)^{1/2}, \quad c = V_1 \left( \frac{1.68\rho_1}{\rho_w} \right)^{1/4}, \quad V_f = \frac{V_1}{3}$$

We finally obtain  $\Delta = 0.35 l$ . Therefore, at approximate consideration, the velocity of the incident flow  $V_1$  and the density ratios  $\rho_1/\rho_w$  influence the value of shock wave recession through the characteristic dimension of the comet only.

In conclusion I express my gratitude to S. A. Kaplan for his guidance in the performance of the work.

\*\*\* THE END \*\*\*

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(\*) [This formula is evidently misprinted in the original text]

continued.../..

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